Supplementary - About the return period of a catastrophe

## Expected return period (RP)

According to section 3.1 of the main paper, two point events and of max-stable associated processes at the line of positive real numbers with expected exceedance frequencies

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can be presented by ( replace in (17) in the main paper)

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We assume that the expectation of the random variable is and hence, the conditional expectation (point event is known). There are the following reasons for these assumptions. The exceedance frequency of points can be understood as a mixture of exceedance frequency of shifted sub processes with events . This means, of is a mixture of by the probability density distribution of positive random variable

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| This can be transformed under consideration of the stochastic definition of an expectation for a positive valued random variable (Upton and Cook, 2008) |  |
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| Since (1) and (2) apply, the following also apply | () |
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We do not claim this would be a complete mathematical proof, but this is the idea of a proof. Condition for this idea of a proof is that the Poisson point processes are infinitely divisible. Furthermore, it is highly likely that this result is already explicitly or implicitly published in a statistical paper or a book about stochastic.

## Validation of the extension of Schlater’s 1st theorem

Schlather’s (2002) 1st theorem is extended in the main paper to a process with marginal Poisson process (9) with exceedance frequency function and corresponding unit Frechet distributed marginal maxima (10). Here the idea of a proof is the following integral for the one-dimensional space

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|  | (8) |

With as the stochastic event magnitude according the main paper and the area function that is a measure for the expected number of points by integral This includes that of Schlathers 1st theorem. With substitution and swap of interval bounds we modify (8) and validate the assumption

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|  | (9) |

This is what we wanted to validate. The validation for is basically the same, however, the substitutions and swaps of interval bounds must be carried out for every dimension. The extension can also be validated heuristically as implicitly done by the Monte-Carlo simulation in Figure S1.

## Combined return period without max-stable dependence

For the dependence instance without max-stability, we can validate heuristically the reproductivity of the average of two RP. We simulate for the process of main paper (9) in a one-dimensional space with probability density function (PDF) of the standard normal distribution (Figure S2a) and (max-stable) and . The distance between the local points is 1. The underlying series of random numbers of the Monte-Carlo simulation is the same for both variants of stability.

a)  b)  c) 

Figure S1: Validation of CRP as average of RP T1 and T2 without max-stable association via Monte Carlos simulation and comparison with max-stable case: a) plot of simulated point events T1 and T2 according to the description of the main paper and for the same series of numbers from a pseudo random generator, b) exceedance frequency of resulting CRP, c) as b) with logarithmic plot of frequency.

## Examples of functions

In the following figure, several possible PDFs are presented which can be applied in Schlather’s (2002) 1st Theorem. They can also be combined with random fields.

a) b)c) d)

Figure S2: Examples for possible functions that can be applied as random function in Schlather’s 1st Theorem with the PDF of the standard normal distribution (broken black line) as reference: a) PDF normal distribution with any standard deviation, b) PDF of a double exponential distribution, c) PDF of a uniform distribution with sites x without an event (Y(x)=0), d) combination of the PDF of the standard normal distribution with a random field with log-normal distributed margins.

## Maximum likelihood method for Gumbel distribution and its adaptation

The well-known Gumbel distribution (Gumbel, 1935 and 1941) has the cumulative distribution function (CDF) with scale parameter and shift (location) parameter

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|  | (10) |

The following estimators (Clarke, 1973) for sample of observations of random variable

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Eq. (11) is solved iteratively before the shift parameter is estimated. To achieve more stability during the iteration algorithm we modify eq. (11) without changing the meaning

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The Gumbel distribution applies for block maxima such as the maximum of all daily observations of a month or a year. Sometimes, the record is not complete, some of the daily records are missing for technical reasons. This affects the estimation for the annual maxima since the sample's block size is smaller than the modelled block maximum. This incompleteness of the records can be considered by replacing by in the iteration (13) with

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The parameter represents the record completeness of the block ( for 100% completeness). The deviation is simple and follows the idea of max-stability of the Gumbel distribution (10) in the univariate sense. Here, only the performance study is presented for validation. The single observations should be based on similar completeness of records to ensure numerical stability. The performance study also shows that the estimate of scale parameter is biased for our sample size and is corrected here with

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In the performance study, a sample of size is generated by a Monte Carlo simulation 100 000 times. The parameter has been estimated with (12-14) and the new samples of parameters are analysed. Sample mean and variance are listed in Table S1. The original parameterization is and . Incompleteness is considered in the simulation with an example with 28 block maxima from blocks with 100% completeness, 6 block maxima from blocks with 90% completeness, and 6 block maxima from blocks with 70% completeness. The generated block maxima are equivalent to a maximum of a sample of Gumbel distributed random variable with and . In the complete case, the maximum considers 10 observations. In the case of 70% completeness, the maximum only considers 7 observations. The variant with incomplete records for the block maxima and without correction results in a relevant bias of the location parameter. The performance of variants with correct records and variants with incomplete records and correction are remarkably similar.

Table S1: Sample mean and variance of point estimates for a Gumbel distributed random variable.

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| **Variant** | **Scale parameter** | | **Location parameter** | |
| **Sample mean** | **Variance** | **Sample mean** | **Variance** |
| **Complete record** | 0.981 | 0.015 | 0.008 | 0.029 |
| **Incomplete record, uncorrected** | 0.991 | 0.016 | -0.064 | 0.028 |
| **Incomplete record, corrected by** (14) | 0.981 | 0.015 | 0.010 | 0.028 |

## Bias reduction of local RP

From point estimates for the Gumbel distribution according to the previous section, the exceedance frequency and RP according to equation (30) of the main paper can be estimated and compared with the actual values of fixed . Even though exceedance frequency and RP are strongly linked via the reciprocal, the performance of their estimates differ considerably. For the study, we used the same samples as in the previous section. The bias and mean square error (MSE) of exceedance frequency is relatively small and is only relevant for exceedingly small RP with high exceedance frequency (Table S2). This is a range that is not important. In contrast, the bias and MSE of RP is large in the important range decades and centuries. We have developed the following correction function for the RP via an optimization

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As listed in Table S2, the corresponding (absolute) bias and MSE of the corrected RP are much smaller now. For the inverse computation, the larger solution of the quadratic equation is used. The correction applies to the unit period of the sample.

Table S2: Bias and MSE of estimated EF and RP via ML method (original distribution of maxima according to standard Gumbel distribution with μ=0 and σ=1, sample size n=40, 100 000 repetitions in Monte Carlo simulation).

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| **Actual values** | | | **Estimated EF uncorrected** | | **Estimated RP uncorrected** | | **Estimated RP corrected** | |
| **z** | **EF** | **RP** | **Bias** | **MSE** | **Bias** | **MSE** | **Bias** | **MSE** |
| 7.60E+00 | 5.00E-04 | 2.00E+03 | 1.21E-04 | 4.34E-07 | 3.84E+03 | 2.72E+09 | -1.26E+01 | 3.06E+07 |
| 6.91E+00 | 1.00E-03 | 1.00E+03 | 1.84E-04 | 1.28E-06 | 1.43E+03 | 1.80E+08 | -3.11E+00 | 4.52E+06 |
| 6.21E+00 | 2.00E-03 | 5.00E+02 | 2.67E-04 | 3.78E-06 | 5.34E+02 | 1.27E+07 | -5.22E-01 | 6.73E+05 |
| 5.30E+00 | 5.00E-03 | 2.00E+02 | 4.00E-04 | 1.57E-05 | 1.47E+02 | 4.55E+05 | 7.25E-02 | 5.56E+04 |
| 4.61E+00 | 1.00E-02 | 1.00E+02 | 4.82E-04 | 4.56E-05 | 5.48E+01 | 4.30E+04 | 6.56E-02 | 8.64E+03 |
| 3.91E+00 | 2.00E-02 | 5.00E+01 | 4.72E-04 | 1.31E-04 | 2.02E+01 | 4.63E+03 | 2.32E-02 | 1.36E+03 |
| 3.22E+00 | 4.00E-02 | 2.50E+01 | 2.40E-04 | 3.68E-04 | 7.28E+00 | 5.44E+02 | -1.72E-03 | 2.14E+02 |
| 2.30E+00 | 1.00E-01 | 1.00E+01 | -6.60E-04 | 1.38E-03 | 1.77E+00 | 3.40E+01 | -1.06E-02 | 1.79E+01 |
| 1.61E+00 | 2.00E-01 | 5.00E+00 | -1.58E-03 | 3.53E-03 | 5.60E-01 | 4.13E+00 | -9.24E-03 | 2.61E+00 |
| 0.00E+00 | 1.00E+00 | 1.00E+00 | 1.86E-02 | 3.18E-02 | 1.18E-02 | 3.19E-02 | -3.10E-03 | 2.83E-02 |
| -6.93E-01 | 2.00E+00 | 5.00E-01 | 8.87E-02 | 1.47E-01 | -6.92E-03 | 6.91E-03 | -1.56E-03 | 6.61E-03 |
| -2.30E+00 | 1.00E+01 | 1.00E-01 | 1.46E+00 | 1.86E+01 | -4.14E-03 | 7.68E-04 | -1.25E-04 | 8.05E-04 |
| -3.69E+00 | 4.00E+01 | 2.50E-02 | 1.19E+01 | 1.22E+03 | -8.53E-04 | 1.10E-04 | 7.78E-05 | 1.21E-04 |

## Non max stable CRP scaling

The scaling of a combined return period (CRP) for European winter storms over Germany is realized by factor in equation (24) of the main paper for the max stable case. This factor applies for the non max stable case only for the relation between and . The scaling of local RP is now approximately realized by factor per station being computed with parameters , , , and per storm

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The parameters are listed in Table S3. The first three parameters have been estimated by a simple regression analysis after a heuristic estimate of parameter . The resulting relation between coefficient of variation (CV) and scaled RP match well with these from analysed observations (Figure 6 e and f of the main paper). Details of the historical storms are also listed in Supplementary Data.

Table S3: Parameters for non max stable scaling of wind fields.

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| **Hist. Storm** | **Parameters** | | | |
| **a** | **b** | **c** | **d** |
| Jennifer | -0.0065 | 0.9056 | 0.0012 | 0.0189 |
| Anna | -0.0034 | 0.9512 | 0.0025 | 0.0134 |
| Jennifer | -0.0031 | 0.9126 | 0.0022 | 0.0466 |
| Nina & Oralie | -0.0043 | 0.9509 | 0.0025 | 0.0083 |
| Dorian (Cyrus) | -0.0031 | 0.9622 | 0.0022 | 0.0080 |
| Kyrill | -0.0070 | 0.8779 | 0.0022 | 0.0300 |
| Emma | -0.0071 | 0.9216 | 0.0020 | 0.0127 |
| Xynthia | -0.0041 | 0.9209 | 0.0033 | 0.0222 |
| Ulli/Andrea | -0.0076 | 0.851 | 0.0012 | 0.0790 |
| Christian | -0.0044 | 0.8304 | -0.0044 | 0.0250 |
| Xaver | -0.0043 | 0.9188 | 0.0027 | 0.0335 |
| Elon und Felix | -0.0044 | 0.9386 | 0.0025 | 0.0200 |
| Niklas | -0.0011 | 0.9705 | 0.001 | 0.0170 |
| Xavier | -0.0054 | 0.9113 | 0.0044 | 0.0150 |
| Herwart | -0.0052 | 0.9413 | 0.0026 | 0.0100 |
| Friederike | -0.0058 | 0.8828 | 0.0036 | 0.0370 |

## Input data and estimates

The considered wind stations of German meteorological service are listed in a sheet of the Supplementary-Excel file including considered weights of area and capital. The estimates of CRP are listed in a sheet of the Supplementary-Excel.

## References

Clarke, R.T. Mathematical models in hydrology. Irrig. Drain. Pap. 19, Food and Agr. Organ. Of the U.N., Rom, 1973.

Coles, S. An Introduction to Statistical Modeling of Extreme Values. Book Series: Springer Series in Statistics, Spinger, 2001.

Gumbel, E.J. Les valeurs extrêmes des distributions statistiques. *Annales de l'Institut Henri Poincaré* **5**, 115–158, 1935.

Gumbel E.J. The return period of flood flows. *The Annals of Mathematical Statistics* **12**, 163–190, 1941.

Schlather, M. Models for Stationary Max-Stable Random Fields. Extremes 5, 33–44, 2002.

Upton, G. and Cook, I. *A dictionary of statistics*. 2nd rev. Ed., Oxford University Press, 2008.