Statistical Supplement of

Estimations of statistical dependence as joint return period modulator of compound events. Part I: storm surge and wave height.

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Contents

S1 Statistical dependence (χ)

The main concept of the so-called dependence measure χ (chi) is related to two or more simultaneously observed variables of interest – such as in our case storm surge and wave height – known as observational pairs. If one variable exceeds a certain extreme (high-impact) threshold, then the value of χ represents the risk that the other variable will also exceed a high-impact threshold as explained in Hawkes (2004), Svensson and Jones (2004a & 2004b), Petroliagkis et al. (2016).

Following Coles et al. (2000), if all of the extreme observations of two variables exceed a given threshold at the same time, this indicates total dependence (χ = 1). If the extreme observations of one variable exceed a given threshold but the second variable does not, this indicates total independence (χ = 0). Similarly, if the extreme observations of one variable exceed a given threshold but the other variable produces lower observations than would normally be expected, this indicates negative dependence (χ = −1). In practice, in tidal and estuarine environments, assessing the probability of flooding from the joint occurrence of both high storm surge and high wave values is not an easy process, as high surges and waves might be related to the same prevailing meteorological conditions (Beersma and Buishand, 2004), thus independence cannot and should not always be assumed. For instance, if we assume independence between input variables, this might underestimate considerably the likelihood of flooding (estimated by the product of their individual probability) resulting in higher risk for the coastal community. Similarly, assuming total dependence could be too conservative. Further, as variables reach their extreme values, special methodologies of estimating statistical dependence could be utilised as the one documented in Buishand (1984).

A brief description of this methodology based on Coles et al. (2000) is described below (Sect. 2) while the basic theory behind the utilisation of an optimal copula function refers to Nelsen (1998), Joe (1997), Currie (1999), Wahl et al. (2015).

S2 Estimation of statistical dependence (χ)

For bivariate random variables (X, Y) with identical marginal distributions, the dependence measure (χ) can estimate the probability of one variable being extreme provided that the other one is extreme:

$$\chi = \lim_{z \to z^*} \Pr\left(Y > z \mid X > z\right) \qquad \text{(S1)}$$

where z* is the upper limit of the observations of the common marginal distribution.

For obtaining identical marginal distributions, each set of observations is ranked separately and each rank is then divided by the total number of observations resulting in a data transformation with Uniform [0, 1] margins. At this point, it is convenient to consider the bivariate cumulative function $F(x, y) = Prob(X \le x, Y \le y)$ that describes the dependence between X and Y completely. The effect of different marginal distributions can be diminished by assuming the copula function C in the domain [0, 1] x [0, 1] such as:

$$F(x, y) = C\{ F_x(x), F_y(y) \} \qquad (S2)$$

where F_x and F_y can be any marginal distributions. Such utilisation of the copula function has the same effect as if observations were ranked separately and divided by the total number of observations. In addition, the copula C contains the complete information about the joint distribution of X and Y and it is invariant to marginal transformation. This means that C is invariant to marginal transformation and it can be described as the joint distribution function of X and Y. Further, X and Y are transformed to new variables U and V with Uniform [0, 1] margins. It follows that the dependence measure $\chi(u)$ for a given threshold u can be given by:

$$\chi(u) = 2 - \frac{\ln \Pr(U \le u, V \le u)}{\ln P(U \le u)} \quad \text{for } 0 \le u \le 1 \qquad (S3)$$

Taken into account the upper limit of the observations (previously defined as z^* in Eq. S1), the dependence measure $\chi(u)$ will be given by:

$$\chi = \lim_{u \to 1} \chi(u) \qquad (S4)$$

Details of deriving Eq. S3 can be found in Coles et al. (2000). Based on Eq. S3, a set of χ values can be evaluated at different quantile levels u. The selection of a particular level u corresponds to threshold levels (x^*, y^*) for the two different data series. For applying Eq. S3, the number of appropriate observation-pairs (X, Y) is counted for estimating the numerator and denominator terms (Eq. S5 & Eq. S6):

$$P(U \le u, V \le u) = \frac{\text{Number of } (X, Y) \text{ such that } X \le x^* \text{ and } Y \le y^*}{\text{Total number of } (X, Y)} \qquad (S5)$$

and

$$\ln P(U \le u) = \frac{1}{2} \ln\left[\frac{\text{Number of } X \le x^*}{\text{Total number of } X} \cdot \frac{\text{Number of } Y \le y^*}{\text{Total number of } Y} \right] \qquad (S6)$$

In this study, a set of routines (mat_chi) based on Matlab software were coded following Eq. S3 to S6 for estimating χ. Additional modules and routines based on the integrated statistical package R were also used for estimating dependence terms and inter-comparing various parameters. Emphasis was given on the routine “taildep” of the module “extRemes” (https://cran.r-project.org/web/packages/extRemes/extRemes.pdf) that is capable of estimating χ values when a critical percentile (extreme) threshold is considered. Another “powerful” routine capable of providing a variety of dependence graphs and plots (besides single estimated values of χ) has been the routine “chiplot” of the module “evd” (Extreme Value Distributions) of R (https://cran.r-project.org/web/packages/evd/evd.pdf). The routine chiplot is also capable of providing confidence intervals at any preselected level.

Besides estimating values of χ, similar routines (mat_chibar) were coded in Matlab following Coles et al. (2000) for calculating the “sister” attribute of χ, namely chibar ($\bar{\chi}$). Chibar (chi_bar) parameter refers to the statistical dependence of asymptotically independent variables whereas chi (χ) refers to the statistical dependence of asymptotically dependent ones. Details on the estimation of chibar are documented in Coles et al. (2000) whereas examples and how to utilise ($\bar{\chi}$) can be found in Coles (2001). The class of asymptotic dependence appears to be the case in Literature, having reached a consensus that there is strong, although not overwhelming, evidence for asymptotic dependence between wave height and surge (Wadsworth et al., 2017).

The concept of asymptotic dependence (χ) is stated with adequate details in Coles et al. (2000). In brief, χ is on the scale [0, 1] with the set (0, 1] corresponding to asymptotic dependence whereas the measure chibar ($\bar{\chi}$) falls within the range [-1, 1] with the set [-1, 1) corresponding to asymptotic independence. That is why the complete pair of χ and $\bar{\chi}$ is required as a summary of extremal dependence:

- $\chi > 0$ & $\bar{\chi} = 1$ reveals asymptotic dependence, in which case the value of χ determines a measure of strength of dependence within the class
- $\chi = 0$ & $\bar{\chi} < 1$ reveals asymptotic independence, in which case the value of $\bar{\chi}$ determines the strength of dependence within the class.

For estimating both χ and $\bar{\chi}$ parameters, the general POT (Peaks-Over-Threshold) methodology was followed. Such an approach (POT) is considered as giving a more accurate estimate of the probability distribution than using the annual maximum series (see details in Stedinger et al., 1993). Applying POT as described in detail in Defra TR1 Report (2005), the selection of an optimal threshold for the data pairs (~2.3 events per year) was adopted as suggested in Defra TR3 Report (2005). Care was taken to force two POT extreme compound events not occurring on consecutive days, but separated by at least three days from each other. Emphasis was also given on the stability of χ (graph) curves identifying the area that dependence was clearly converging to a specific value (no abrupt fluctuations).

Relatively small differences among various estimates made by chiplot of evd (R), taildep of extRemes (R) and mat_chi (Matlab) were found. This most probably is due to the unavoidable dissimilarities between the criteria being imposed on data pairs when applying POT methodology (selection of different critical thresholds).

S3 Selection of critical thresholds

For selecting a threshold u (referring to a critical percentile) as required in Eq. S3, it seems appropriate to transform the Uniform distribution to an annual maximum non-exceedance probability scale (Defra TR3 Report, 2005). Then the annual maximum non-exceedance probability (α) is defined as:

$$\alpha = \text{Prob (Annual maximum} \leq x) \qquad \text{(S7)}$$

where x is the magnitude of the source variable. Such non-exceedance probability relates to the return period, T_α, as:

$$T_\alpha = 1 / (1 - \alpha) \qquad \text{(S8)}$$

For a transformation from annual maximum to POT series (see details and scope in the previous Sect. S2), we define the “new” non-exceedance probability, the so-called p, referring to a rate of λ events per year, relating to the annual maximum of Eq. S7, as:

$$\alpha = \exp(-\lambda (1 - p)) \qquad \text{(S9)}$$

where 1-p is the “new” exceedance probability of the POT series. The term (1 – p) can be estimated graphically (Hazen, 1914) leading to Equation S10:

$$\lambda (1 - p) = (N_e / N) * (i - 0.5) / N_e = (i - 0.5) / N \qquad \text{(S10)}$$

where i, represents the rank of the independent POT events, N_e is the number of POT events while N represents the number of years (see details in Defra TR3 Report, 2005). The independence criterion of two POT events to be separated by at least three days (six half-day intervals in the max12 case) was applied for all river ending points. Combining Eq. S9 and Eq. S10, an estimation of α is possible as given by Eq. S11:

$$\alpha = \exp(-(i - 0.5) / N) \qquad \text{(S11)}$$

Therefore, going after the magnitude of x in Eq. S7 is equivalent as trying to define the magnitude of the POT element with rank i in Eq. S11 for the same maximum non-exceedance annual probability, alpha (α). After the selection of an optimal threshold (u) based on alpha (α), the estimation of χ is straightforward (Eq. S3). The main idea here is to use χ in a relatively simple formula that also uses as input the individual return periods T_X and T_Y for estimating the joint return period ($T_{X,Y}$), like the formula described by Eq. S12 following White (2007), Australian Rainfall & Runoff Project 18 (2009).

$$T_{XY} = \sqrt{T_X * T_Y \,/\, \chi^2} \qquad \text{(S12)}$$

Studying Eq. S12 closely it becomes obvious that dependence is capable of substantially modulating the joint return period. For details and potential limitations of Eq. S12, see discussions in White (2007), Hawkes (2004), Meadowcroft et al. (2004), Australian Rainfall & Runoff Project 18 (2009). In cases of totally dependent variables, Eq. S12 yields the common individual return period of source variables as an estimation of the joint return period. An example of how to utilise Eq. S12 is given in Sect. 4.2 of the main text for the river ending point of Rhine (NL). Further, some limitations of Eq. S12 could be overcome if a more complete formula is used such as Eq. 2.15 for instance taken from White's thesis (2007) but this is above the scope of the current study.

S4 Significance

The values of dependence (χ) corresponding to the 5% significance level were estimated using a permutation method as described by Good (1994). As in Defra TR3 Report (2005), 199 permutations of the data were made for each surge-wave pair and a new value of χ was calculated each time. All 199 values of χ were subsequently ranked in descending order and the 5% significance level was defined by selecting the 10th largest value representing the 95% point of the null distribution (the hypothetical distribution occurring if data-pairs were indeed independent). Care was taken to preserve the seasonality since permutation of data was performed by randomly reshuffling intact blocks of one year time period.

It should be kept in mind that the significance level of 5% represents the probability of rejecting the null hypothesis when it is true. In simple words, it indicates a 5% risk of concluding that a difference exists capable of rejecting the null hypothesis (the population mean equals to the hypothesized mean) when there is no actual difference.

S5 Confidence intervals

For the estimation of confidence intervals, a well-tested bootstrapping method was applied similar to the permutation method already used for estimating significance (for details see Defra TR3 Report, 2005). This bootstrapping resulted in the generation of many new data-sets (resamples). The original sample of observation-pairs was used as the main (reference) distribution from which the resamples were chosen randomly. A large number of data sets were generated for calculating χ for each of these new data sets. This provided a sample of what would occur for a range of situations. Seasonality was kept intact by sampling in blocks of one year, rather than using individual observation-pairs. The balanced resampling as documented by Fisher (1993) was applied ensuring that each year occurs equally often overall among the total number of bootstrap samples. In total, 199 bootstrap samples of the data were made for each station-pair and a new χ value was calculated each time. The 199 values were subsequently ranked in descending order and the 10 and 190 largest values were accepted as determining the 90% confidence interval.

To draw the distinction between significance (previous Sect. S4) and confidence levels it should be noted that a confidence interval is a range of values that is likely to contain an unknown population parameter (in our case the statistical dependence) whereas the significance represents the probability of rejecting the null hypothesis when it is true. It follows that if a random sample is drawn many times, a certain percentage of the confidence intervals will contain the population mean. That is the reason behind the usage of confidence intervals for bounding the mean or standard deviation.

S6 Selection of critical thresholds resulting in the consideration of top-80 events

Extreme value analysis can be carried out using two types of data series (Bezak et al., 2014), annual maximums (MA) or flows above a certain threshold (POT) for Peak Over Threshold. The POT model used in this study can be composed of the Poisson, binomial and negative binomial distributions for modelling the annual number of events above threshold, and of exponential or generalized Pareto distributions for magnitudes of exceedances.

Since values of dependence (χ) can be estimated for any lower or upper threshold, initial trials were performed studying the behaviour of χ over a wide range of thresholds. Findings were similar to those contained in Defra TR3 Report (2005), justifying the selection of an optimal threshold for “alpha” (α) equal to 0.1 corresponding to an annual maximum being exceeded in 9 out of 10 years (see details in Sect. S3). This value (0.1) of alpha was considered for both mat_chi (χ) and mat_chibar ($\bar{\chi}$) routines when utilising POT (Peaks-Over-Threshold) methodology resulting in an annual maximum of ~2.3 compound events.

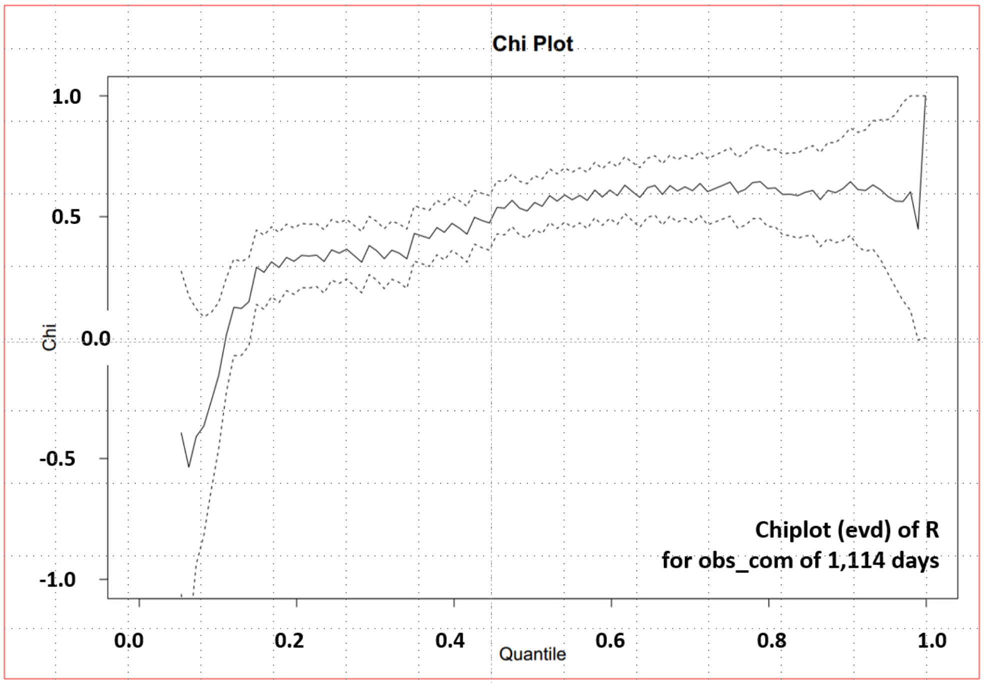
Such annual threshold of ~2.3 events corresponds to the top 80 (Top-80) compound events taking place during any (POT separated) day of the total 12,753 days and it was dictated mainly by two factors: the threshold had to be low enough to allow a sufficient number of data points to exceed it for estimating dependence reliably, while being high enough for the data points to be regarded as extremes.

S7 Details and examples of the statistical packages used in the study

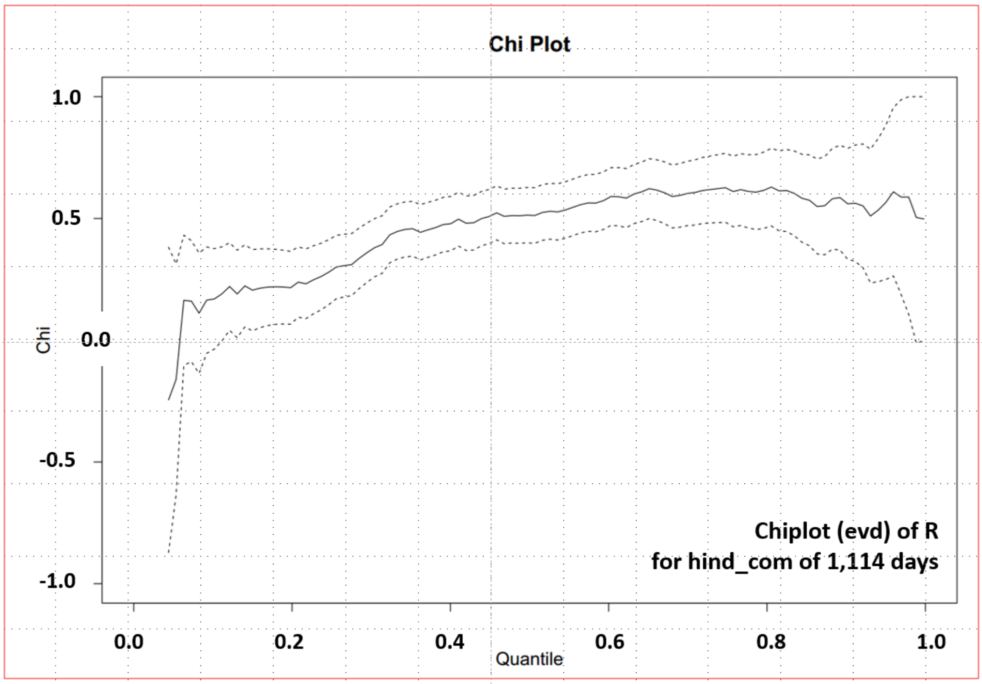
In this study, a set of routines (mat_chi) based on Matlab software were coded following Eq. S3 to S6 for estimating χ. Additional modules and routines based on the integrated statistical package R were also used for estimating dependence terms and inter-comparing various parameters. Emphasis was given on the routine "taildep" of the module "extRemes" (https://cran.r-project.org/web/packages/extRemes/extRemes.pdf) that is capable of estimating χ values when a critical percentile (extreme) threshold is considered.

Another "powerful" routine capable of providing a variety of dependence graphs and plots (besides single estimated values of χ) has been the routine "chiplot" of the module "evd" (Extreme Value Distributions) of R (https://cran.r-project.org/web/packages/evd/evd.pdf). The routine chiplot is also capable of providing confidence intervals at any preselected level. As mentioned above (Sect. S2) relatively small differences among various estimates made by chiplot of evd (R), taildep of extRemes (R) and mat_chi (Matlab) were found and this most probably is due to the unavoidable dissimilarities between the criteria being imposed on data pairs when applying POT methodology.

Examples of estimated statistical dependence (χ) values between surge (HvH) and wave (LiG) max24 values in obs_com (upper panel), hind_com (middle panel) and in hind_tot (lower panel) mode by chiplot routine of evd module (R) are given in Fig. S1.



Chi Plot
1.0
0.5
0.0
-0.5
-1.0
Chi
0.0
0.2
0.4
0.6
0.8
1.0
Quantile
Chiplot (evd) of R
for obs_com of 1,114 days



Chi Plot
1.0
0.5
0.0
-0.5
-1.0
Chi
0.0
0.2
0.4
0.6
0.8
1.0
Quantile
Chiplot (evd) of R
for hind_com of 1,114 days

5

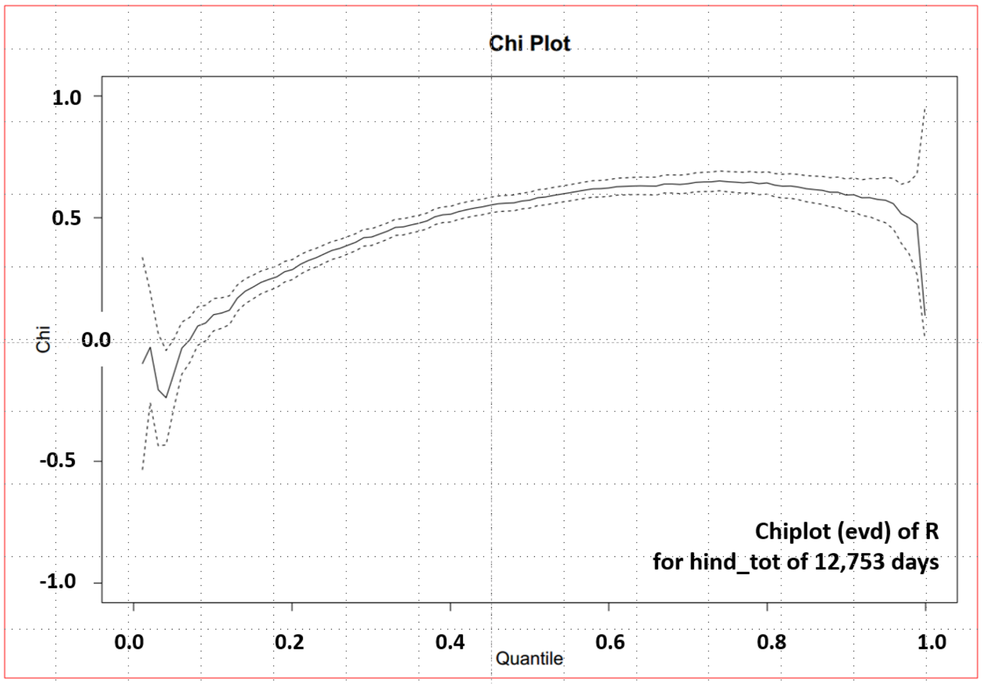


Figure S1. Estimated χ values between surge (HvH) and wave (LiG) max24 values in c (upper panel) & hind_com (middle panel) and in hind_tot (lower panel) mode by chiplot routine of evd module (R).

Studying closely Fig. S1 it becomes obvious that considerable high values of dependence are estimated over all three (obs_com, hind_com & hind_tot) modes. The importance and implications of such high values of dependence can be demonstrated with an example by considering the total hindcast (hind_tot) series for surge (HvH) and wave (LiG). Utilising the Matlab function "gevfit" an estimation of the return levels having a 100-year return period for surge and wave height variables was made (1.78 and 6.05 metres respectively). Inserting the common return period value (100-year) together with the estimated χ value (0.56) in Eq. S12, the Joint Return Period (JRP) of such a compound event (surge ≥ 1.78 metres and significant wave height ≥ 6.05 metres) was estimated at ~179 years.

Such a value (~179 years) is significantly different from the value of 10,000 years representing the estimated JRP assuming that surge and wave variables were totally independent. In a case like this (of independent events), the dependence would have been equal to zero and the JRP would be given by the product of their individual probabilities (Blank, 1982).

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