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Damage functions for climate-related hazards: unification and uncertainty analysis

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1 Variance-based sensitivity analysis

1.1 Additional Sensitivity Indices

A variance-based sensitivity analysis allows to explain output variance by the contributions from explanatory variables (Saltelli et al., 2008). The general method is described in Sec. 5.1 of the main paper. For the estimation of the first-order effect we do not employ the estimator recommended by Saltelli et al. (2010), which appears to be robust only for variables with zero mean. Instead, we use the corresponding Jansen estimator (Jansen, 1999; Saltelli et al., 2010) whose results rapidly converge with increasing sample size s. Hence, the first-order effects index FO_i is given by

$$\mathrm{FO}_i = 1 - \frac{\frac{1}{2s}\sum_{k=1}^{s}\left(\mathcal{F}(\mathbf{B})_k - \mathcal{F}(\mathbf{C}_{(i)})_k\right)^2}{\sigma^2}, \tag{1}$$

with

$$\sigma^2 = \frac{1}{2s}\sum_{k=1}^{s}\left(\mathcal{F}(\mathbf{A})_k^2 + \mathcal{F}(\mathbf{B})_k^2\right) - \left(\frac{1}{s}\sum_{k=1}^{s}\mathcal{F}(\mathbf{A})_k + \mathcal{F}(\mathbf{B})_k\right)^2. \tag{2}$$

The extension to higher-order effects (interactions) is straight-forward. For second-order effects, we construct matrix $\mathbf{C}_{(i,j)}$ such that we take all columns $l \notin \{i,j\}$ from $\mathbf{A}$ and all columns $l \in \{i,j\}$ from $\mathbf{B}$. To estimate the second-order effect index $\mathrm{SO}_{i,j}$, we replace $\mathbf{C}_{(i)}$ with $\mathbf{C}_{(i,j)}$ in Eq. (1) and subtract first-order effects of input variables i and j. It follows that

$$\mathrm{SO}_{i,j} = 1 - \frac{\frac{1}{2s}\sum_{k=1}^{s}\left(\mathcal{F}(\mathbf{B})_k - \mathcal{F}(\mathbf{C}_{(i,j)})_k\right)^2}{\sigma^2} - \mathrm{FO}_i - \mathrm{FO}_j \tag{3}$$

for $i \neq j$.

Similarly, we calculate the third-order effect index $\mathrm{TO}_{i,j,k}$ by constructing the corresponding matrix $\mathbf{C}_{(i,j,k)}$ and subtracting low-order terms,

$$\mathrm{TO}_{i,j,k} = 1 - \frac{\frac{1}{2s}\sum_{k=1}^{s}\left(\mathcal{F}(\mathbf{B})_k - \mathcal{F}(\mathbf{C}_{(i,j,k)})_k\right)^2}{\sigma^2} - \mathrm{FO}_i - \mathrm{FO}_j - \mathrm{FO}_k - \mathrm{SO}_{i,j} - \mathrm{SO}_{i,k} - \mathrm{SO}_{j,k} \tag{4}$$

for $i \neq j \neq k \neq i$.

It is clear from inspection, that Eqs. (3) and (4) simplify considerably for models with only two and three random variables, respectively. For models with only two random variables, the second-order effect index becomes

$$\mathrm{SO}_{1,2} = 1 - \mathrm{FO}_1 - \mathrm{FO}_2. \tag{5}$$

Similarly, for models with three random variables the third-order effect index simplifies to

$$\mathrm{TO}_{1,2,3} = 1 - \mathrm{FO}_1 - \mathrm{FO}_2 - \mathrm{FO}_3 - \mathrm{SO}_{1,2} - \mathrm{SO}_{1,3} - \mathrm{SO}_{2,3}. \tag{6}$$

1.2 Additional sensitivity results for the Lisbon case study

Figure 1 shows the first- and second-order effect indices of intrinsic and extrinsic (hazard threshold) uncertainties [cf. Fig. 6(c) of the main paper]. The interaction seen for flood levels below $0.5\,\mathrm{m}$ is due to the fact that the uncertainty in the hazard threshold determines the occurrence of a damage at such low flood levels and that, consequently, the intrinsic uncertainties are conditional on the occurrence of a damaging event.

Figure 2 shows the first-, second-, and third-order effect indices for the intrinsic uncertainties in both the microscale and the macroscale damage function. If compared with the total-effects index [cf. Fig. 6(a-b) of the main paper], it is seen that the first-order effects are the dominant contribution to the total effects index. Panels (c) and (d) show that there is some interaction between the variation in asset value and the uncertainty of the threshold exceedance. However, this interaction is limited to inundation levels below $1\,\mathrm{m}$ and only contributes lightly to the overall model variance (less than 0.2 of output variance). Figure 2(e-f) show that third-order effects are negligible.

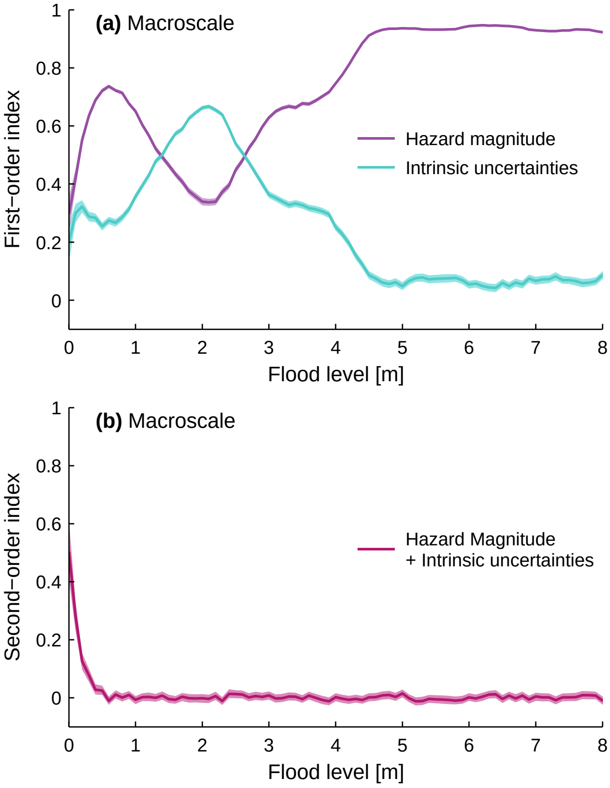

Figure 1. Results of the sensitivity analysis of the macroscale damage function for the Lisbon case study, relating the joint effect of intrinsic uncertainties to the effect of uncertainty in the hazard threshold. Panel **(a)** shows the direct, first-order effect, while panel **(b)** shows the second-order effect due to interaction between the uncertainty sources.

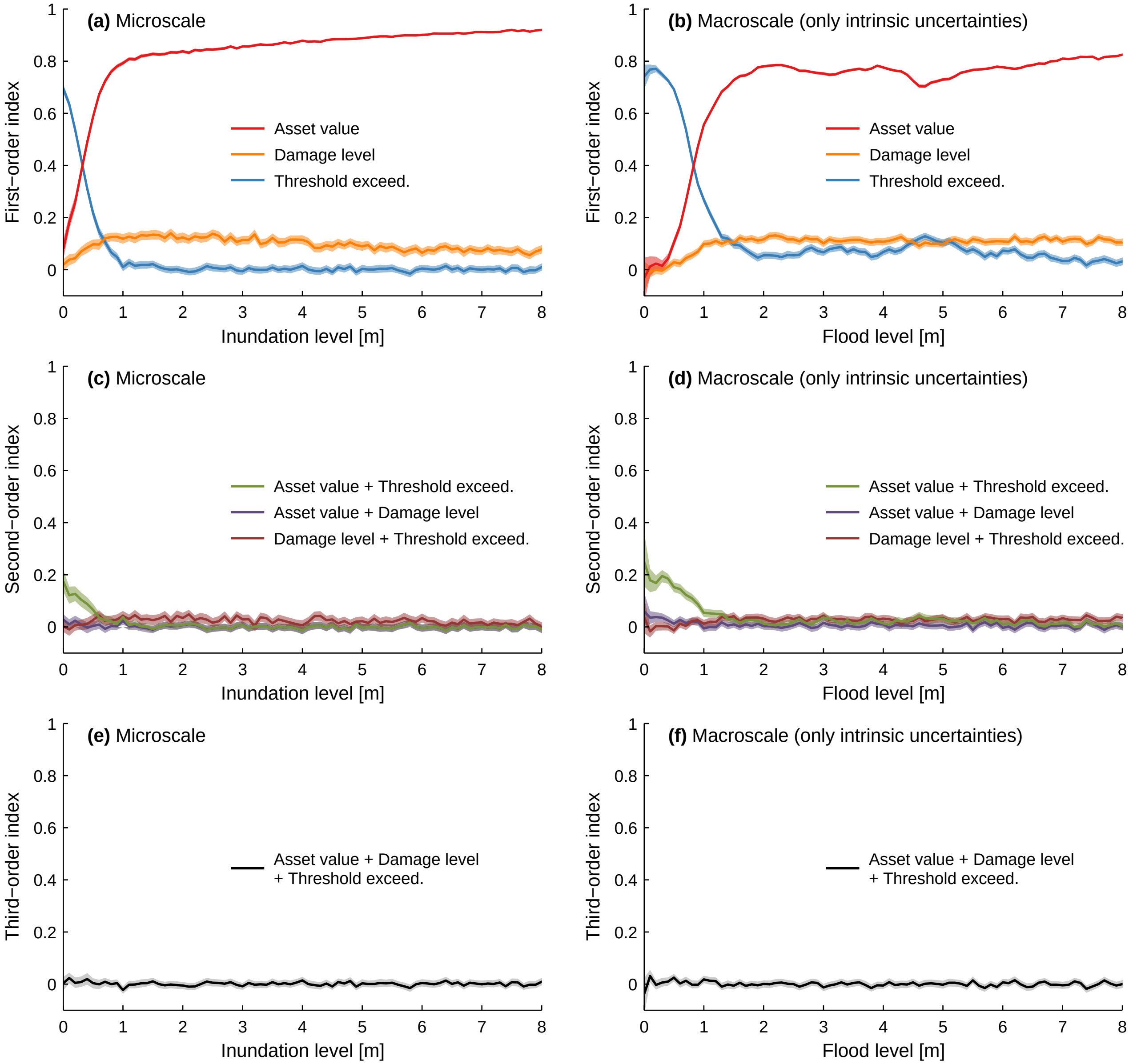

Figure 2. Results of the sensitivity analysis of the microscale **(a,c, and e)** and the macroscale **(b, d, and f)** damage function for the Lisbon case study, taking into account only intrinsic uncertainties. Each column comprises the first-, second-, and third-order effects of the respective uncertainty sources on the output variance. First-order effects are directly attributable to a source of uncertainty, while higher-order effects arise from interactions between two or more uncertain variables.

1.3 Additional sensitivity results for the German storm damage case study

Figure. 3 shows the first- and second-order effect indices of intrinsic and extrinsic (hazard threshold) uncertainties within the macroscale damage function [cf. Fig. 7(c) of the main paper]. There is some interaction at low hazard magnitudes, analogous to the Lisbon case study (cf. Fig. 1).

Figure 4 shows the first-, second-, and third-order effect indices for the intrinsic uncertainties in both the microscale and the macroscale damage function [cf. Fig. 7(a-b) of the main paper]. The same reasoning as for the Lisbon case study applies (cf. Fig. 2).

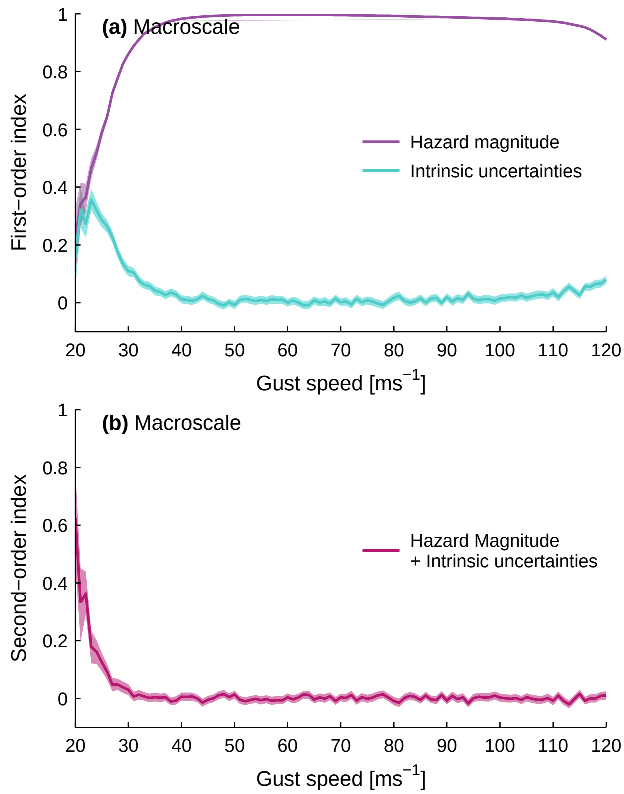

Figure 3. Results of the sensitivity analysis of the macroscale damage function for the German storm-damage case study, relating the joint effect of intrinsic uncertainties to the effect of uncertainty in the hazard threshold. Panel **(a)** shows the direct, first-order effect, while panel **(b)** shows the second-order effect due to interaction between the uncertainty sources.

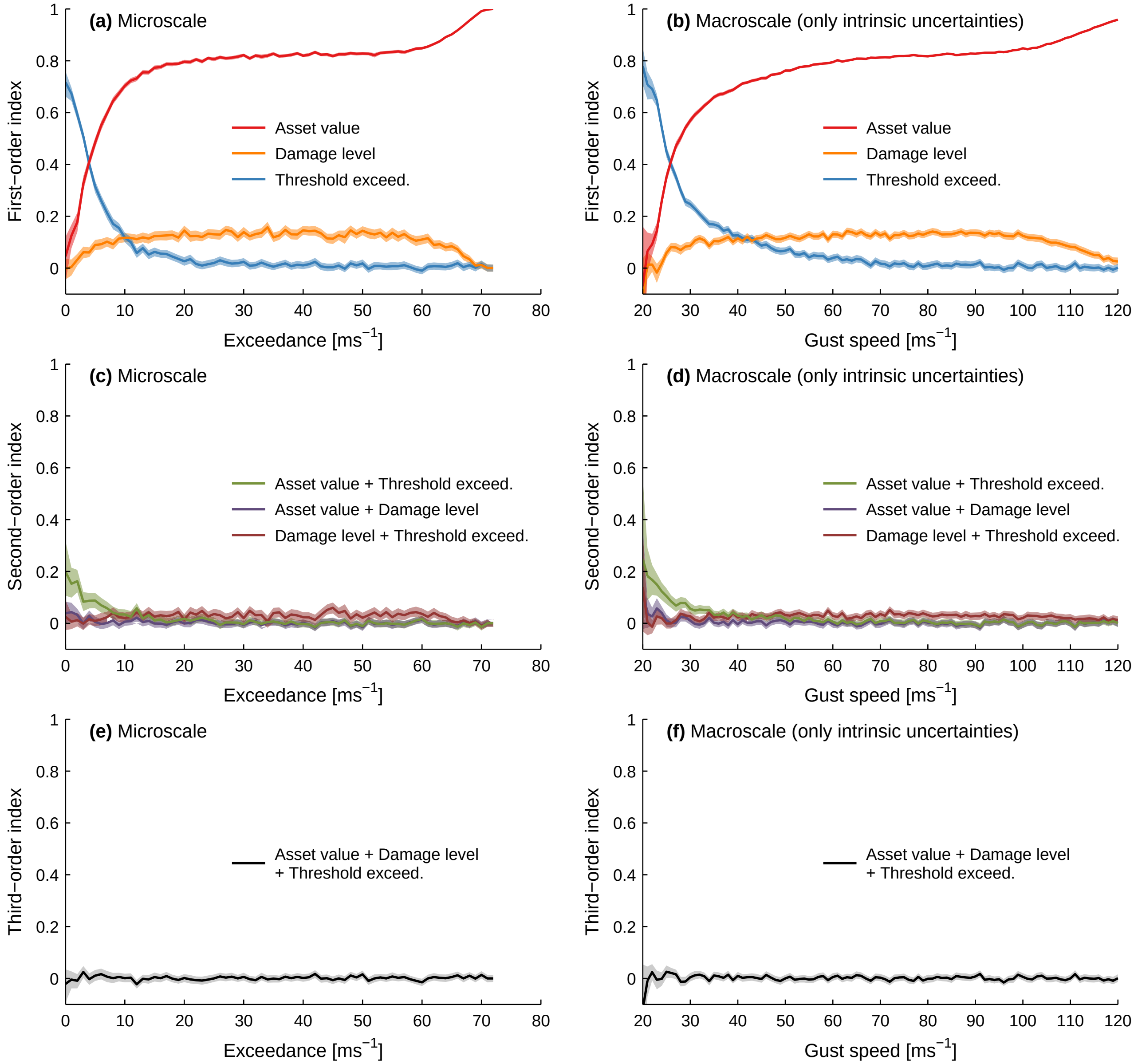

Figure 4. Results of the sensitivity analysis of the microscale **(a,c, and e)** and the macroscale **(b, d, and f)** damage function for the German storm-damage case study, taking into account only intrinsic uncertainties. Each column comprises the first-, second-, and third-order effects of the respective uncertainty sources on the output variance. First-order effects are directly attributable to a source of uncertainty, while higher-order effects arise from interactions between two or more uncertain variables.

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